Exact multisoliton solutions of the higher-order nonlinear Schrödinger equation with variable coefficients

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We investigate the generalized higher-order nonlinear Schrödinger equation with variable coefficients under two sets of parametric conditions. The exact one-soliton solution is presented by the ansatz method for one set of parametric conditions. For the other, exact multisoliton solutions are presented by employing the Darboux transformation based on the Lax pair. As an example, we consider a soliton control system, and the results show that the soliton control system may relax the limitations to parametric conditions. The stability of the solution is discussed numerically; the results reveal that finite initial perturbations, such as amplitude, chirp, or white noise, could not influence the main character of the solution. In addition, the evolution of a quite arbitrary Gaussian pulse and the interaction between neighboring pulses have been studied in detail.

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Propagation of femtosecond light pulses in optical fibers is of particular interest because of their extensive applications to telecommunication and ultrafast signal-routing systems. The problem can be described by the higher-order nonlinear Schrödinger (HNLS) equation, which was derived by Kodama and Hasegawa [1,2]. The best known solutions of the HNLS equation are those for solitary waves or solitons under certain parametric conditions. In recent years, many authors have analyzed the HNLS equation from different points of view and some interesting results have also been obtained [3–17].

The concept of soliton control is a new and important development in the application of solitons. Picosecond soliton control, which is described by the nonlinear Schrödinger (NLS) equation with variable coefficients, has been extensively studied theoretically because of its potential value [18–26]. We would note the fact that the first soliton dispersion management experiment in a fiber with hyperbolically decreasing group velocity dispersion was realized as early as in 1991 by Bogatyrev *et al.* [27]; therefore, a study of the NLS-type equation with variable coefficients is significant. It has been discussed in previous papers [28–30]. However, to our knowledge, studies of femtosecond soliton control have not been widespread. The problem is governed by the HNLS equation with variable coefficients as follows:

$$iq_z = -D_2(z)q_{tt} - 2R(z)|q|^2q + iD_3(z)q_{ttt} + i\alpha(z)(|q|^2q)_t + if(z)q(|q|^2)_t + i\Gamma(z)q, \tag{1}$$

where $q(z,t)$ is the complex envelope of the electrical field in a comoving frame, $D_2(z)$ and $D_3(z)$ represent the group velocity dispersion and third-order dispersion, respectively, $R(z)$ is the nonlinearity parameter, and the parameters $\alpha(z)$ and $f(z)$ are related to self-steepening and delayed nonlinear response effects, respectively. $\Gamma(z)$ denotes the amplification or absorption coefficient. They are real functions of the normalized propagation distance z, and t is the retarded time. Equation (1) describes short pulse propagation in weakly dispersive and nonlinear dielectrics with distributed parameters. In particular, when $D_3(z)=0$, $\alpha(z)=0$, and $f(z)=0$, Eq. (1) can reduce to the NLS equation with variable coefficients.

In this paper, we investigate the generalized higher-order nonlinear Schrödinger equation with variable coefficients under two sets of parametric conditions. The exact one-soliton solution is presented by the ansatz method for one set of parametric conditions. For the other, multisoliton solutions are presented by employing the simple, straightforward Darboux transformation based on the Lax pair, and particularly the exact one- and two-soliton solutions in explicit forms are generated. The importance of the results presented here is twofold. First, exact multisoliton solutions to the generalized HNLS equation with variable coefficients under certain parametric conditions are obtained in a simple way. The finding of a mathematical algorithm to discover soliton solutions in nonlinear dispersive systems with spatial parameter variations is helpful for future research. Second, these results are useful not only in the design of transmission lines with soliton management, but also in some experiments of other problems, such as femtosecond lasers.

Generally, Eq. (1) is not integrable. To solve Eq. (1), we begin our analysis by assuming a solution given by the expression [24]

$$q_1(z,t) = \eta_1(z)\sqrt{\frac{D_2}{R}}e^{i\phi_1}\text{sech }\theta_1, \tag{2}$$

where $\theta_1=\eta_1(z)[t+\rho(z)]$, $\phi_1=\xi_1(z)t+\omega(z)$, and $\eta_1(z)$, $\rho(z)$, $\xi_1(z)$, and $\omega(z)$ are related to the inverse width, the group velocity, the frequency shift, and the phase of the pulse,

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which are functions of propagation distance z, respectively. Substituting the ansatz (2) into Eq. (1), removing the exponential term, and then separating the real and imaginary parts, we can obtain two sets of solvable conditions as follows:

$$\Gamma(z) = \frac{1}{2}\frac{RD_{2,z} - D_2R_z}{RD_2}, \tag{3}$$

$$\eta_1 = \text{const}, \tag{4}$$

$$6D_3R = (3\alpha + 2f)D_2, \tag{5}$$

with

$$\text{(i)}\ \xi_1 = 0, \tag{6}$$

or

$$\text{(ii)}\ \xi_1 = \text{const}, \quad \alpha + f = 0. \tag{7}$$

In the case (i), one can write Eq. (2) in the form

$$q_1 = \eta_1\sqrt{\frac{D_2}{R}}\exp\left[i\eta_1^2\int_0^z D_2(\zeta)d\zeta\right] \times \text{sech}\left[\eta_1\left(t + \eta_1^2\int_0^z D_3(\zeta)d\zeta\right)\right]. \tag{8}$$

The existence of soliton solution (8) is the result of weak balance among third-order dispersion, self-steepening, and delayed nonlinear response effects described by Eq. (5). From the expression (8) one can clearly see that the velocity of the soliton is determined by $\eta_1^2D_3(z)$, the phase shift is related to $\eta_1^2D_2(z)$, and the amplification or absorption is determined by the relation (3). Thus we may obtain the optimal control system by choosing the distributed parameters $D_3(z)$, $D_2(z)$, and $R(z)$ for each specific problem, appropriately.

In the case (ii), by employing Ablowitz-Kaup-Newell-Segur technology we can construct the linear eigenvalue problem for Eq. (1) as follows:

$$\Psi_t = U\Psi, \quad \Psi_z = V\Psi, \tag{9}$$

where $\Psi = (\varphi_1, \varphi_2)^T$, T represents the transpose of the matrix, and U and V can be given by

$$U = \lambda J + P, \quad V = \begin{pmatrix} A & B \\ C & -A \end{pmatrix}$$

with

$$J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P = \sqrt{\frac{R}{D_2}}\begin{pmatrix} 0 & q \\ -\overline{q} & 0 \end{pmatrix},$$

$$A = 4D_3\lambda^3 + 2iD_2\lambda^2 + 2\frac{RD_3}{D_2}|q|^2\lambda + \frac{RD_3}{D_2}(\overline{q}q_t - q\overline{q_t}) + iR|q|^2,$$

$$B = \sqrt{\frac{R}{D_2}}\left[4D_3q\lambda^2 + 2(D_3q_t + iD_2q)\lambda + D_3\left(q_{tt} + 2\frac{R}{D_2}q|q|^2\right) + iD_2q_t\right],$$

$$C = \sqrt{\frac{R}{D_2}}\left[-4D_3\overline{q}\lambda^2 + 2(D_3\overline{q_t} - iD_2\overline{q})\lambda + D_3\left(-\overline{q_{tt}} - 2\frac{R}{D_2}\overline{q}|q|^2\right) + iD_2\overline{q_t}\right],$$

where the overbar represents the complex conjugate, and λ is the spectral parameter which is a complex constant. It is easy to verify that Eq. (1) can be recovered by the compatibility condition $U_z - V_t + [U,V] = 0$. In particular, when Eq. (1) is with constant coefficients, the conditions (5) and (7) are reduced to the Hirota conditions, Eq. (9) represents the Lax pair of the Hirota equation, and the corresponding results have been given in [5,32]. In the following, we investigate Eq. (1) by employing a simple, straightforward Darboux transformation based on the linear eigenvalue problem (9) [31–33].

According to the standard procedure of Darboux transformation [32,33], we can obtain the fundamental Darboux transformation

$$q_1 = q + 2\sqrt{\frac{D_2}{R}}\frac{(\lambda_1 + \overline{\lambda_1})\varphi_1\overline{\varphi_2}}{|\varphi_1|^2 + |\varphi_2|^2}, \tag{10}$$

and analogous to this procedure and taking the Darboux transformation n times, we find the following formula:

$$q_n = q + 2\sqrt{\frac{D_2}{R}}\sum_{m=1}^{n}\frac{(\lambda_m + \overline{\lambda_m})\varphi_{1,m}(\lambda_m)\overline{\varphi_{2,m}}(\lambda_m)}{A_m}, \tag{11}$$

where

$$\varphi_{k,m+1}(\lambda_{m+1}) = (\lambda_{m+1} + \overline{\lambda_m})\varphi_{k,m}(\lambda_{m+1}) - \frac{B_m}{A_m}(\lambda_m + \overline{\lambda_m})\varphi_{k,m}(\lambda_m),$$

$$A_m = |\varphi_{1,m}(\lambda_m)|^2 + |\varphi_{2,m}(\lambda_m)|^2,$$

$$B_m = \varphi_{1,m}(\lambda_{m+1})\overline{\varphi_{1,m}}(\lambda_m) + \varphi_{2,m}(\lambda_{m+1})\overline{\varphi_{2,m}}(\lambda_m),$$

$m = 1, 2, \ldots, n$, $k = 1, 2$, and $(\varphi_{1,1}(\lambda_1), \varphi_{2,1}(\lambda_1))^T$ is the eigenfunction of Eq. (9) corresponding to λ_1 for q. Substituting the zero solution $q = 0$ of Eq. (1) into Eq. (11), we can systematically obtain multisoliton solutions for Eq. (1). Here we present only one- and two-soliton solutions in explicit forms.

By setting $n = 1$ in Eq. (11) and taking the complex spectral parameter $\lambda_1 = (\eta_1 + i\xi_1)/2$, we find that the one-soliton solution is of the form

$$q_1 = \eta_1\sqrt{\frac{D_2(z)}{R(z)}}e^{i\phi_k}\text{sech}\,\theta_k, \tag{12}$$

where

$$\theta_k = \eta_k\left(t + (\eta_k^2 - 3\xi_k^2)\int_0^z D_3(\zeta)d\zeta - 2\xi_k\int_0^z D_2(\zeta)d\zeta\right) - \theta_{k0},$$

$$\phi_k = \xi_k t + \xi_k(3\eta_k^2 - \xi_k^2)\int_0^z D_3(\zeta)d\zeta + (\eta_k^2 - \xi_k^2)\int_0^z D_2(\zeta)d\zeta - \phi_{k0}, \qquad (13)$$

$k=1$, and θ_{10} and ϕ_{10} are arbitrary real constants. In the above two expressions, the real part η_1 of the spectral parameter λ_1 is mainly dependent on the pulse width and its imaginary part ξ_1 describes the frequency shift. The phase shift is related to both real and imaginary parts of the spectral parameter λ_1, and the initial position and the initial phase of the soliton are determined by the parameters θ_{10} and ϕ_{10}, respectively. From the soliton solution (12), we find that the velocity of the soliton is determined by $(\eta_1^2-3\xi_1^2)D_3(z) -2\xi_1 D_2(z)$, which depends on the distributed parameters $D_2(z)$ and $D_3(z)$ except for the spectral parameter λ_1. Thus, we can control the velocity of the soliton by managing the distributed parameters $D_2(z)$ and $D_3(z)$ in optical soliton communication systems [26]. In particular, when $D_2(z) =R(z)=\text{const}$, $D_3(z)=\text{const}$, and $\xi_1=0$, the one-soliton solution (12) can be reduced to a simple form, as shown in Ref. [18].

When $n=2$, from Eq. (11), we can find

$$q_2 = \sqrt{\frac{D_2(z)}{R(z)}}\frac{G}{F} \qquad (14)$$

where

$$G = a_1\cosh\theta_2 e^{i\phi_1} + a_2\cosh\theta_1 e^{i\phi_2} + ia_3(\sinh\theta_2 e^{i\phi_1} - \sinh\theta_1 e^{i\phi_2}),$$

$$F = b_1\cosh(\theta_1 + \theta_2) + b_2\cosh(\theta_2 - \theta_1) + b_3\cos(\phi_2 - \phi_1),$$

with

$$a_k = \frac{\eta_k}{2}[\eta_k^2 - \eta_{3-k}^2 + (\xi_1 - \xi_2)^2],$$

$$b_k = \frac{1}{4}\{[\eta_1 + (-1)^k\eta_2]^2 + (\xi_1 - \xi_2)^2\},$$

$$a_3 = \eta_1\eta_2(\xi_1 - \xi_2), \quad b_3 = -\eta_1\eta_2.$$

θ_k and ϕ_k are given in Eq. (13) with $k=1$, 2, where we have used $\lambda_k=(\eta_k+i\xi_k)/2$. Based on the exact solution (14), we can conveniently analyze the transmission properties of two femtosecond optical solitons in inhomogeneous systems. From the expression of θ_k, one can clearly see that the velocity of each soliton in the two-soliton solution (14) is determined by $(\eta_k^2-3\xi_k^2)D_3(z)-2\xi_k D_2(z)$, which is dependent not only on the spectral parameters λ_1 and λ_2, but on the distributed parameters $D_3(z)$ and $D_2(z)$. Thus, we can trap the velocity of each soliton to form a bound-soliton solution by designing the distributed parameters $D_3(z)$ and $D_2(z)$.

For example, we consider a soliton control system with the group velocity dispersion parameter

$$D_2(z) = d_2\exp(-gz), \qquad (15)$$

the nonlinearity parameter

$$R(z) = r\exp(-\sigma z), \qquad (16)$$

and the third-order dispersion parameter

$$D_3(z) = d_3\exp(-hz), \qquad (17)$$

where r and σ are the parameters to describe the nonlinearity, and d_2 and g, d_3 and h are related to the group velocity dispersion and third-order dispersion, respectively. The presented system is similar to the one given by Eq. (14) of Ref. [26]. In this situation, the gain/loss distributed function is of the form $\Gamma(z)=(\sigma-g)/2$ ($\sigma>g$ for the gain; $\sigma<g$ for the loss), and the velocity of each soliton in the two-soliton solution (14) can be written as $V_k=d_3(\eta_k^2-3\xi_k^2)\exp(-hz) -2d_2\xi_k\exp(-gz)$. From it one can see that V_k for $h>0$ and $g>0$ tends to zero when z goes to infinity. This means that when $h>0$ and $g>0$ the bound-soliton solution can always be formed, and is independent of the spectral parameters λ_1 and λ_2. Figure 1(a) presents the separating evolution plot of the two-soliton solution given by Eq. (14) for $h>0$ and $g>0$. From Fig. 1(a) we can see that the separation between the two solitons [Eq. (14)] keeps constant except for the change of the soliton velocity at the beginning of propagation. When $hg<0$ or $h<0$ and $g<0$, V_k tends to $+\infty$ or $-\infty$ (depending on the choice of the spectral parameters λ_1 and λ_2) as z goes to infinity, and hence the two-soliton solution given by Eq. (14) may describe the elastic collision between solitons. Figure 1(b) shows the procedure of the elastic collision for $h<0$ and $g>0$. The results show that one may control the interaction between the pulses by choosing the parameters h and g appropriately.

It is worth noting that the existence of soliton solutions (12) and (14) depends on the specific nonlinear and dispersive features of the medium, which have to satisfy the conditions (3)–(5) and (7), i.e., Hirota conditions in a more general sense, which could be called the generalized Hirota conditions. These constraint conditions present the strict balances among third-order dispersion, self-steepening, and delayed nonlinear response effects. In real applications, however, it may be difficult to produce exactly such balances. Therefore a study for nongeneralized Hirota conditions is necessary. Here we take the solution (12) and the soliton control system given by Eqs. (15)–(17) as an example, and perturb the generalized Hirota conditions in two ways: (a) $\alpha=6D_3(z)R(z)/D_2(z)$, $\alpha+f=D_3(z)$; and (b) $\alpha =7D_3(z)R(z)/D_2(z)$, $\alpha+f=0$. The results are shown in Fig. 2. We clearly see that, after a short adjustment, the soliton approaches the stable state. In fact, we have made more numerical simulations for the nongeneralized Hirota conditions, and the results show that the solution is still stable and the evolution of the soliton is not sensitive to perturbed conditions. Therefore, we may infer that the soliton control system presented here may relax the limitations to parametric con-

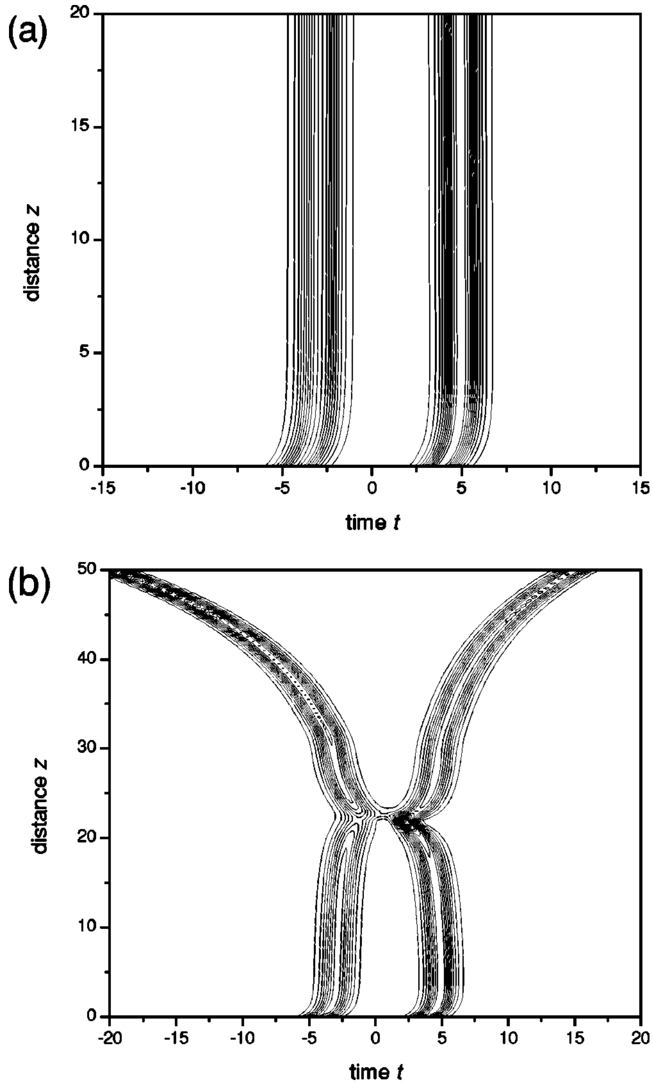

FIG. 1. (a) The separating evolution plot of soliton solution given by Eq. (14) for the system parameters $d_2=g=1$, $d_3=h=0.1$, $r=1$, $\sigma=g$. The other parameters adopted are $\eta_1=-1$, $\eta_2=1.1$, $\xi_1=\xi_2=0.6$, $\theta_{10}=\theta_{20}=1$, $\phi_{10}=\phi_{20}=0$. (b) The elastic collision for the system parameters $h=-0.1$, and the other parameters the same as in (a).

ditions. This may make the soliton control technique more realistic and leave scope for more physical explanations and applications in the future.

To demonstrate stability with respect to finite perturbations for the solutions, we still take Eq. (12) as an example and perform various types of numerical experiments. The results reveal that finite initial perturbations (10%), such as amplitude, frequency, and white noise, could not influence the main character of the solution. Figure 3 presents the evolution plot of an initial perturbed sech pulse $1.1\eta_1\sqrt{D_2(0)/R(0)}\exp(0.9i\xi_1 t-\theta_{10})[\mathrm{sech}(\eta_1 t-\phi_{10})+0.1\,\mathrm{random}(t)]$ for the system given by Eqs. (15)–(17). In addition, we investigated the problem of whether the solution is stable under more general conditions. Figure 4 shows the evolution of an initial Gaussian pulse $q(0,t)=\exp(-t^2)$. From this plot, we find that the solution is still stable except for

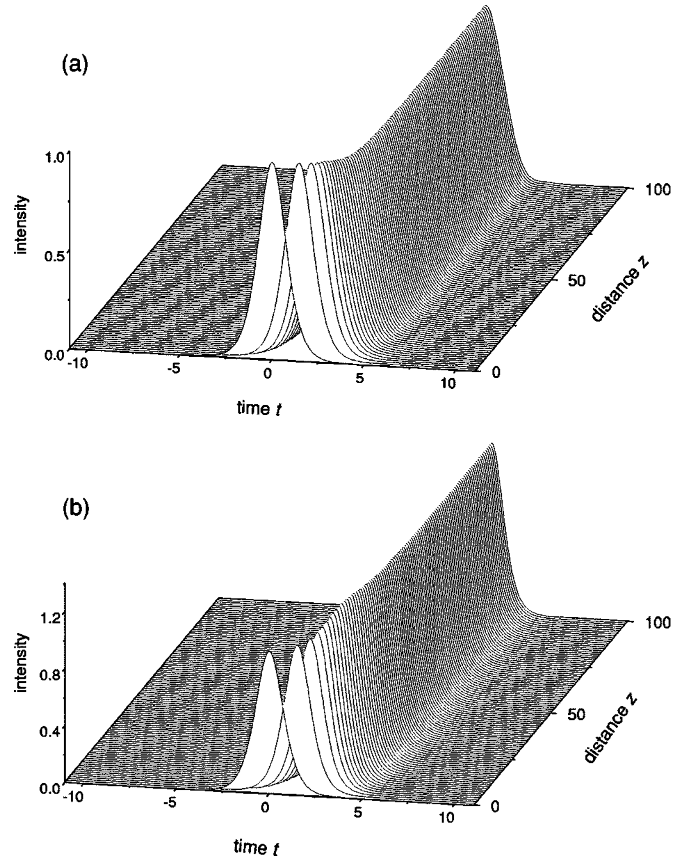

FIG. 2. The evolution plot of soliton solution given by Eq. (12) for the parameter $\sigma=g$. Here the parameters adopted are $\eta_1=\xi_1=1$, $\theta_{10}=\phi_{10}=0$. (a) $\alpha=6D_3(z)R(z)/D_2(z)$, $\alpha+f=D_3(z)$; (b) $\alpha=7D_3(z)R(z)/D_2(z)$, $\alpha+f=0$.

some oscillation attached to the solitons' tails.

Finally, we investigated interaction between neighboring pulses. Here, we consider two cases: (a) the initial pulse is of Gaussian type with equal amplitude in the form $q(0,t)=\exp[-(t+T_0/2)^2]+\exp[-(t-T_0/2)^2]$; (b) the initial pulse is of sech type with equal amplitude in the form $q(0,t)=\mathrm{sech}(t+T_0/2)+\mathrm{sech}(t-T_0/2)$, where T_0 accounts for the

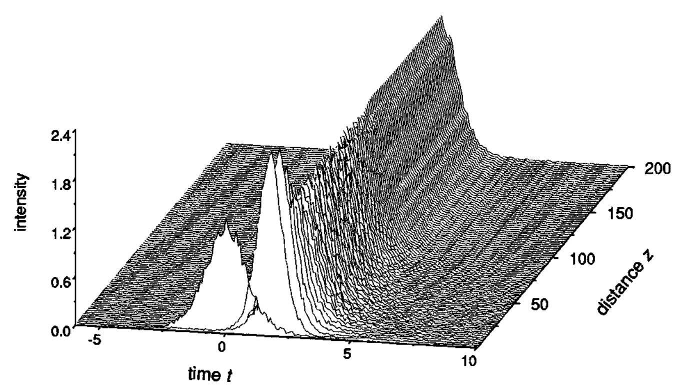

FIG. 3. The evolution plot of an initial perturbed sech pulse $1.1\eta_1\sqrt{D_2(0)/R(0)}\exp(0.9i\xi_1 t-\theta_{10})[\mathrm{sech}(\eta_1 t-\phi_{10})+0.1\,\mathrm{random}(t)]$ for the system given by Eqs. (15)–(17), where the system parameters are $d_2=g=1$, $d_3=h=0.1$, $r=1$, $\sigma=g$, and the other parameters are $\eta_1=\xi_1=1$, $\theta_{10}=\phi_{10}=0$.

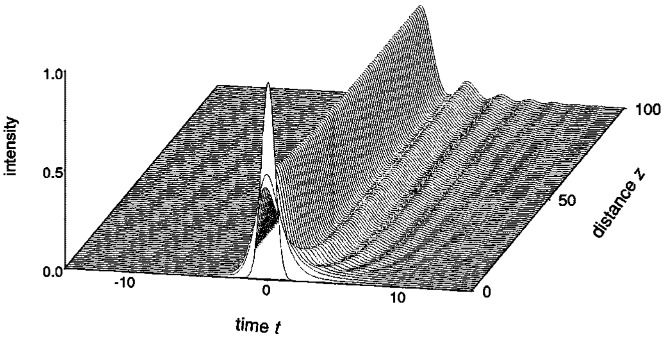

FIG. 4. The evolution of an initial Gaussian pulse $q(0,t)=\exp(-t^2)$ for the system parameters $d_2=g=1$, $d_3=h=0.1$, $r=1$, $\sigma=g$.

pulse separation. Figures 5(a) and 5(b) present the separating evolution plot of the neighboring Gaussian-type pulse and the neighboring sech-type pulse under the same pulse separation $T_0=7$ for the parameter $\sigma=g$, respectively. From Fig. 5 we can see that for the Gaussian-type pulse the interaction between pulses gives rise to unequal amplitude except for some oscillation attached to the solitons' tails, as shown in Fig. 5(a); for the sech-type pulse, however, the separation of the neighboring solitons keeps constant, and is smaller than that for the ideal nonlinear Schrödinger equation [34], as shown in Fig. 5(b). Therefore, we may infer that the combined effects of controlling both the group velocity dispersion distribution and the nonlinearity distribution can restrict the interaction between the neighboring solitons to some extent. It is advantageous to increase the information bit rate in optical soliton communications.

In conclusion, we have considered the higher-order nonlinear Schrödinger equation with variable coefficients under two sets of parametric conditions. We have presented the exact one-soliton solution by the ansatz method for one set of parametric conditions. For the other, we have presented multisoliton solutions by employing the Darboux transformation, and the exact one- and two-soliton solutions in explicit forms have been generated. The solutions are of general application in short pulse propagation in weakly dispersive and nonlinear dielectrics. Furthermore, as an example, we have given a soliton control system, and the results have shown that the soliton control system presented here may relax the limitations to parametric conditions. The stability of the solution has been discussed numerically. The results have revealed that finite initial perturbations, such as

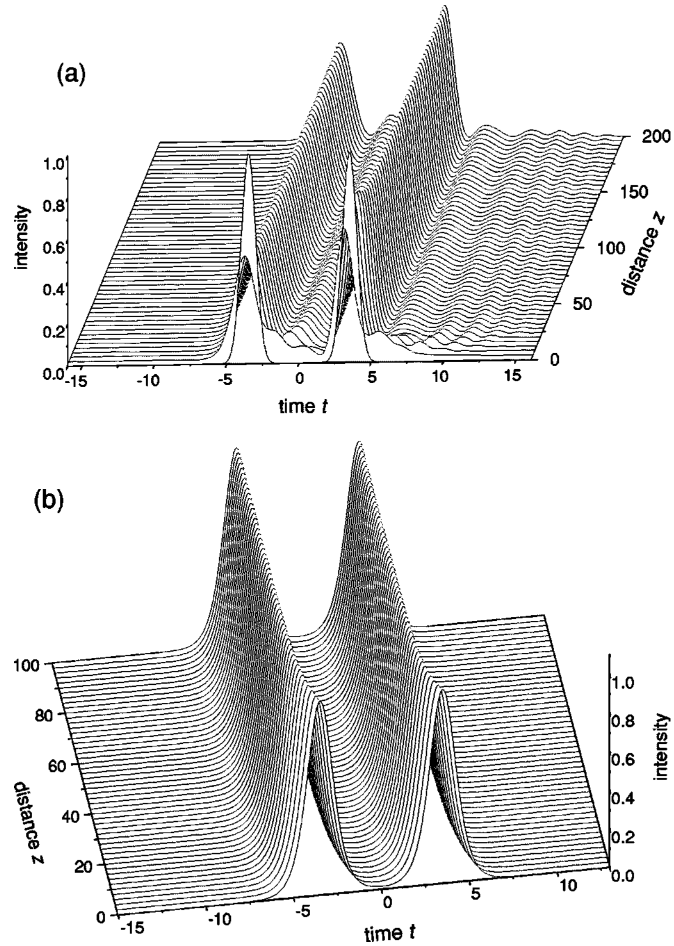

FIG. 5. The evolution plot of the interaction between neighboring pulses with the separation $T_0=7$. Here the system parameters are the same as in Fig. 4. (a) The neighboring Gaussian-type pulse; (b) the neighboring sech-type pulse.

amplitude, frequency, or white noise, could not influence the main character of the solution. In addition, the evolution of a quite arbitrary Gaussian pulse and the interaction between neighboring pulses have been studied in detail. The application of these results to femtosecond optical soliton propagation in long optical fibers should be an interesting task.

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